NOTES AND CORRESPONDENCE

Analytical Solution of Surface Layer Similarity Equations

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ABSTRACT

Turbulent exchange between the surface and the atmosphere strongly depends on the stability of the surface layer. If surface radiometric temperature, rather than aerodynamic temperature, is used to parameterize the surface turbulent fluxes, the solution of the stability parameter is related to the thermal roughness length z_T, which is generally not identical to the aerodynamic roughness length z_0. This note derives the exact solution of the stability parameter equation for a stable surface layer and proposes an approximate analytical solution for an unstable surface layer. The solution can improve the computational efficiency of flux parameterization and is applicable in a wide range of z/z_0 (50–10^4) and z_0/z_T (from less than 1 to greater than 10^4).

1. Introduction

The interaction between the land and the atmosphere is realized through the turbulent exchange of momentum, heat, and moisture at the ground surface, so flux parameterization in the atmospheric surface layer plays an important role in studies of the hydrological cycle and weather prediction. Monin–Obukhov similarity theory describes the mean flow and turbulent characteristics in a horizontally homogeneous surface layer and has been widely applied to the parameterization in the surface layer. In general, the atmospheric surface layer consists of an inertial sublayer and a roughness sublayer (Garratt 1992). The inertial sublayer is well removed from the surface and is thus little affected by individual roughness elements. The resulting flow is horizontally homogeneous, and the similarity theory is applicable to this layer. In contrast, the flow in the roughness sublayer is affected by individual roughness elements, and thus the applicability of the similarity theory to this layer is questionable.

The parameterization of turbulent fluxes requires meteorological data for at least two levels. In numerical simulations of the hydrological cycle, the data at the higher level (or reference level) are provided by an atmospheric model or observation, and the data at the lower level (or ground) are provided by the energy and moisture budget at the surface. Because the similarity theory is not applicable to near-surface flow, the aerodynamic roughness length z_0 and the thermal roughness length z_T are introduced to connect surface variables and the stability in the inertial sublayer.

In theory, z_0 can be expected to be greater than z_T, given that momentum transport is generally more effective than heat transport because of the influence of pressure fluctuations (Mahrt 1996). Over the sea, z_0 is comparable to z_T; over natural homogeneous vegetated surfaces, z_0 is 1 order of magnitude higher than z_T; over a surface with bluff roughness elements, the ratio of z_0 to z_T may be very large (Garratt 1992; Beljaars and Holtslag 1991; Kondo 1994). Therefore, z_0/z_T varies over a wide range.

To parameterize the surface fluxes, it is essential to solve the stability parameter equation, which can be derived from profile functions. For a stable surface layer, it will be shown that it is easy to solve accurately the equation based on linear profile functions. For an unstable surface layer, however, the equation is highly nonlinear, and it is difficult to achieve its exact solution; thus numerical iteration is usually used to solve the equation (e.g., Hanna and Paine 1989; Uno et al. 1995; Lo 1996). To improve computational efficiency, various approximate analytical solutions have been proposed in the past 20 years to replace the iterative method. The approximate solutions of Louis (1979), Byun (1990), and Lee (1997) were based on the assumption that z_0 is equal to z_T. Although the assumption is not valid in general, it is still often used in models such as the Pennsylvania State University–National Center for Atmospheric Research Fifth-Generation Mesoscale Model (Grell et al. 1995) and the Advanced Regional Predic-

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tion System of the University of Oklahoma (Xue et al. 1995). In recent years, a few approximate solutions have been proposed for z_0 not equal to z_T. Mascart (1995) modified the solution of Louis (1979) in a simple way to allow for unequal values of z_T and z_0, but the solution for z_0/z_T less than 10 is not as accurate as that for z_0/z_T greater than 200. Launianen (1995) presented a semianalytical solution for z_0 from 10^{-5} to 10^{-1} m and z_0/z_T in the range of 0.5–7.3. Van den Hurk and Holtslag (1997) extended the solution to be applicable to a higher range of z_0/z_T (>500).

This note proposes a new analytical solution for a common realistic ratio of z_0/z_T. For a stable surface layer, Byun's (1990) solution has been extended from $z_0 = z_T$ to $z_0 \neq z_T$ to obtain the exact solution of the stability parameter equation. For an unstable surface layer, the authors have analyzed the dependence of the stability parameter on $\ln(z/z_0)$, $\ln(z/z_T)$, and the bulk Richardson number and have proposed a simple but relatively accurate solution for several profile forms.

2. Stability parameter equation

From similarity theory, the dimensionless wind shear and potential temperature gradient in a horizontally homogeneous surface layer are usually expressed as

$$\frac{kz}{u_*}\frac{\partial U}{\partial z} = \phi_m(\xi) \quad \text{and} \tag{1a}$$

$$\frac{kz}{\theta_*}\frac{\partial \Theta}{\partial z} = \phi_h(\xi), \tag{1b}$$

where U (m s^{-1}) is wind speed, Θ (K) is potential temperature, u_* (m s^{-1}) is friction velocity, θ_* (K) is temperature scale (or friction temperature), $\xi \equiv z/L$ is stability parameter, $\phi_m(\xi)$ and $\phi_h(\xi)$ are profile functions corresponding to wind and potential temperature, z is the height above the zero-plane displacement, and $L \equiv (T_0 u_*^2)/(kg\theta_*)$ is buoyancy length. Here, T_0 (K) is a representative temperature in the surface layer, k is von Kármán's constant, and g (m s^{-2}) is the acceleration of gravity.

After introduction of the roughness length, Eqs. (1a) and (1b) can be integrated from the roughness length level to the reference level, which results in

$$\frac{U}{u_*} = \frac{1}{k}\left[\ln\frac{z}{z_0} - \psi_m(\xi, \xi_0)\right], \quad \text{and} \tag{2a}$$

$$\frac{(\Theta - \Theta_s)}{\theta_*} = \frac{\mathrm{Pr}_0}{k}\left[\ln\frac{z}{z_T} - \psi_h(\xi, \xi_T)\right], \tag{2b}$$

where Θ_s is the surface radiometric potential temperature, $\xi_0 \equiv z_0/L$, $\xi_T \equiv z_T/L$,

$$\psi_m(\xi, \xi_0) = \int_{\xi_0}^{\xi} \frac{1 - \phi_m(\xi)}{\xi}\, d\xi, \quad \text{and}$$

$$\psi_h(\xi, \xi_T) = \int_{\xi_T}^{\xi} \frac{1 - \phi_h(\xi)/\mathrm{Pr}_0}{\xi}\, d\xi.$$

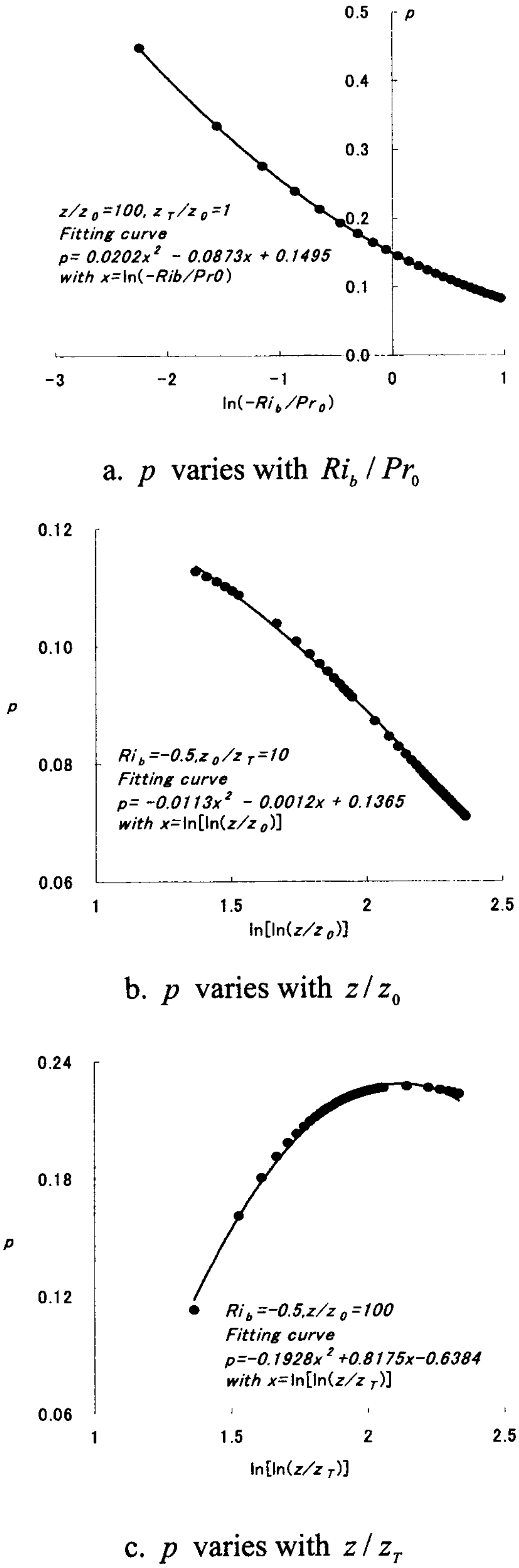


FIG. 1. The variation of modifying factor p with $\ln(-\mathrm{Ri}_b/\mathrm{Pr}_0)$, $\ln[\ln(z/z_0)]$, and $\ln[\ln(z/z_T)]$, respectively. Dots are numerical values of p, and the line is the fitting curve to the dots. Högström (1996) functions are applied to calculate numerical value of p.

TABLE 1. Coefficients in five typical forms of profile functions.

Profile form*	β_m	β_h	γ_m	γ_h	Pr_0 Stable	Pr_0 Unstable	k
B71	4.7	6.4	15.0	9.0	0.74	0.74	0.35
D74	5.0	5.0	16.0	16.0	1.0	1.0	0.41
W80	6.9	9.2	22.0	13.0	1.0	1.0	0.41
DB82	—	—	28.0	14.0	1.0	1.0	0.40
H96	5.3	8.0	19.0	11.6	1.0	0.95	0.40

* Sources: B71: Businger et al. (1971); D74: Dyer (1974); W80: Wieringa (1980); DB82: Dyer and Bradley (1982); H96: Högström (1996).

Given wind speed and temperature at the reference level and at the ground level, it is useful and desirable to define a bulk Richardson number to relate the stability with the mean variables:

$$\mathrm{Ri}_b \equiv \frac{g(z - z_0)}{U^2}\frac{(\Theta - \Theta_s)}{T_0}. \tag{3}$$

By substituting Eqs. (2a) and (2b) into Eq. (3), we get the stability parameter equation:

$$\frac{\mathrm{Ri}_b}{\mathrm{Pr}_0} = \frac{(\xi - \xi_0)[\ln(z/z_T) - \psi_h(\xi, \xi_T)]}{[\ln(z/z_0) - \psi_m(\xi, \xi_0)]^2}. \tag{4}$$

Stability parameter ξ apparently depends on profile functions $\phi_m(\xi)$ and $\phi_h(\xi)$, which are generally expressed as

$$\phi_m = \begin{cases} 1 + \beta_m\xi & \xi > 0 \\ (1 - \gamma_m\xi)^{-1/4} & \xi < 0 \end{cases} \quad \text{and} \tag{5a}$$

$$\phi_h = \begin{cases} \mathrm{Pr}_0(1 + \beta_h\xi) & \xi > 0 \\ \mathrm{Pr}_0(1 - \gamma_h\xi)^{-1/2} & \xi < 0, \end{cases} \tag{5b}$$

where β_m, β_h, γ_m, γ_h, and Pr_0 are coefficients. Coefficient Pr_0 may be different for stable and unstable surface layers. These values in five typical profile forms are shown in Table 1.

3. Solution of stability parameter equation

a. Solution for a stable surface layer

For a stable surface layer,

$$\psi_m(\xi, \xi_0) = -\beta_m(\xi - \xi_0) \quad \text{and} \tag{6a}$$

$$\psi_h(\xi, \xi_T) = -\beta_h(\xi - \xi_T). \tag{6b}$$

By incorporating Eqs. (6a) and (6b) into Eq. (4), one can readily obtain a quadratic equation,

$$\frac{\mathrm{Ri}_b}{\mathrm{Pr}_0} = \frac{(\xi - \xi_0)[\ln(z/z_T) + \beta_h(\xi - \xi_T)]}{[\ln(z/z_0) + \beta_m(\xi - \xi_0)]^2}. \tag{7}$$

Its exact solution is

$$\xi = (-b - \sqrt{b^2 - 4ac})/2a, \tag{8}$$

where

$$a = \frac{\mathrm{Ri}_b}{\mathrm{Pr}_0}\beta_m^2\left(1 - \frac{z_0}{z}\right)^2 - \beta_h\left(1 - \frac{z_0}{z}\right)\left(1 - \frac{z_T}{z}\right), \tag{9a}$$

$$b = \left(2\frac{\mathrm{Ri}_b}{\mathrm{Pr}_0}\beta_m \ln\frac{z}{z_0} - \ln\frac{z}{z_T}\right)\left(1 - \frac{z_0}{z}\right), \quad \text{and} \tag{9b}$$

$$c = \frac{\mathrm{Ri}_b}{\mathrm{Pr}_0}\left(\ln\frac{z}{z_0}\right)^2. \tag{9c}$$

The exact solution is based on linear profile functions. Because the linear functions suppress turbulent exchange too strongly for a very stable surface layer, the solution is only applicable to a moderately stable layer. For a very stable layer, Beljaars and Holtslag (1991) proposed a nonlinear profile form, and van den Hurk and Holtslag (1997) obtained an approximate solution of the stability parameter equation based on the profile form.

b. Solution for an unstable surface layer

For an unstable surface layer, $\psi_m(\xi, \xi_0)$ and $\psi_h(\xi, \xi_T)$ are complicated:

$$\psi_m(\xi, \xi_0) = 2\ln\left(\frac{1 + x}{1 + x_0}\right) + \ln\left(\frac{1 + x^2}{1 + x_0^2}\right) - 2\tan^{-1}x + 2\tan^{-1}x_0 \quad \text{and} \tag{10a}$$

$$\psi_h(\xi, \xi_T) = 2\ln\left(\frac{1 + y}{1 + y_T}\right), \tag{10b}$$

with $x = (1 - \gamma_m\xi)^{1/4}$, $x_0 = (1 - \gamma_m\xi z_0/z)^{1/4}$, $y = (1 - \gamma_h\xi)^{1/2}$, $y_T = (1 - \gamma_h\xi z_T/z)^{1/2}$.

Substitution of Eqs. (10a) and (10b) into Eq. (4) leads to a highly nonlinear equation. Although it is very difficult to find its exact solution, an attempt is made to find an approximate analytical solution that avoids costly numerical iteration. The procedures to explore the solution are 1) find the analytical solution in a slightly unstable range, that is, $-1 \ll \xi < 0$; 2) then extend the solution from $-1 \ll \xi < 0$ to a general unstable range with a modifying factor; and 3) finally determine the modifying factor.

For $-1 \ll \xi < 0$, the profile functions in Eqs. (5a) and (5b) can be approximated as

$$\phi_m \approx 1 + \gamma_m\xi/4, \qquad \phi_h \approx \mathrm{Pr}_0(1 + \gamma_h\xi/2). \tag{11}$$

The form is similar to that for a stable surface layer, and thus ξ is obtained through Eq. (8) if β_m and β_h in Eqs. (9a)–(9c) are replaced with $\gamma_m/4$ and $\gamma_h/2$, respectively. With some simplification, one can derive

$$\xi = \left\{\frac{\mathrm{Ri}_b}{\mathrm{Pr}_0}\frac{[\ln(z/z_0)]^2}{\ln(z/z_T)}\left(\frac{z}{z - z_0}\right)\right\} \div \left[1 - \frac{\mathrm{Ri}_b}{\mathrm{Pr}_0}\frac{\gamma_m^2}{8\gamma_h}\frac{(1 - z_0/z)}{(1 - z_T/z)}\right]. \tag{12}$$

Equation (12) is valid only for $-1 \ll \xi < 0$. For higher instability, a similar solution form is assumed, with a modifying factor p in the denominator:

$$\xi = \left\{\frac{\mathrm{Ri}_b}{\mathrm{Pr}_0}\frac{[\ln(z/z_0)]^2}{\ln(z/z_T)}\left(\frac{z}{z - z_0}\right)\right\} \div \left[1 - \frac{\mathrm{Ri}_b}{\mathrm{Pr}_0}\frac{\gamma_m^2}{8\gamma_h}\frac{(1 - z_0/z)}{(1 - z_T/z)}p\right]. \tag{13}$$

Because ξ in Eq. (4) depends on $\mathrm{Ri}_b/\mathrm{Pr}_0$, $\ln(z/z_0)$, and $\ln(z/z_T)$, it is reasonable to assume that $p = p[\mathrm{Ri}_b/\mathrm{Pr}_0, \ln(z/z_0), \ln(z/z_T)]$. To construct the expression for p, it is necessary to investigate its variation with $\mathrm{Ri}_b/\mathrm{Pr}_0$, $\ln(z/z_0)$, and $\ln(z/z_T)$, respectively. As an example, Fig. 1a shows that the value of p changes with respect to $\ln(-\mathrm{Ri}_b/\mathrm{Pr}_0)$, where $z/z_0 = 100$ and $z_0/z_T = 1$. The dot represents the value derived from Eq. (13) after the iterative solution of ξ is obtained with Eqs. (4) and (10). The figure shows that a quadratic polynomial provides an approximate fit to the dots. Figures 1b,c similarly, show that the variation of p with $\ln[\ln(z/z_0)]$ and with $\ln[\ln(z/z_T)]$ almost follows quadratic polynomial curves if the other two variables are fixed. Therefore, the quadratic polynomial below is used to approximate p:

$$p \approx \sum c_{ijk}[\ln(-\mathrm{Ri}_b/\mathrm{Pr}_0)]^i\{\ln[\ln(z/z_0)]\}^j\{\ln[\ln(z/z_T)]\}^k, \tag{14}$$

where i, j, and $k = 0$, 1, and 2, and $i + j + k \leq 2$; c_{ijk} are coefficients that depend on profile functions. Based on a numerical solution of ξ, the coefficients in Eq. (14) corresponding to five typical profile forms have been regressed and are shown in Table 2.

In summary, the stability parameter ξ can be calculated analytically from Eq. (8) for a moderately stable layer and from Eq. (13) for an unstable layer. In the special case of $z_0 = z_T$, the current solution of Eq. (8) is identical to the previous analytical solution for stable layers [see Eq. (19) of Byun (1990) and Eqs. (11) and (12) of Lee (1997)]; for unstable layers, the solution of Eq. (13) is similar to the analytical solution of Lee (1997) but includes the dependence of coefficients on the Richardson number.

4. Verification of analytical solution for unstable surface layer

The turbulent fluxes from a ground surface may be calculated as follows:

$$\tau = \rho C_D U^2, \tag{15a}$$

$$H = -\rho C_H U(\Theta - \Theta_s), \quad \text{and} \tag{15b}$$

$$E = -\rho C_H U(q - q_s), \tag{15c}$$

where τ is surface stress, H is sensible heat flux, E is evaporation rate, Θ_s is potential temperature, and q_s is specific humidity at the surface. In addition, U, Θ, and

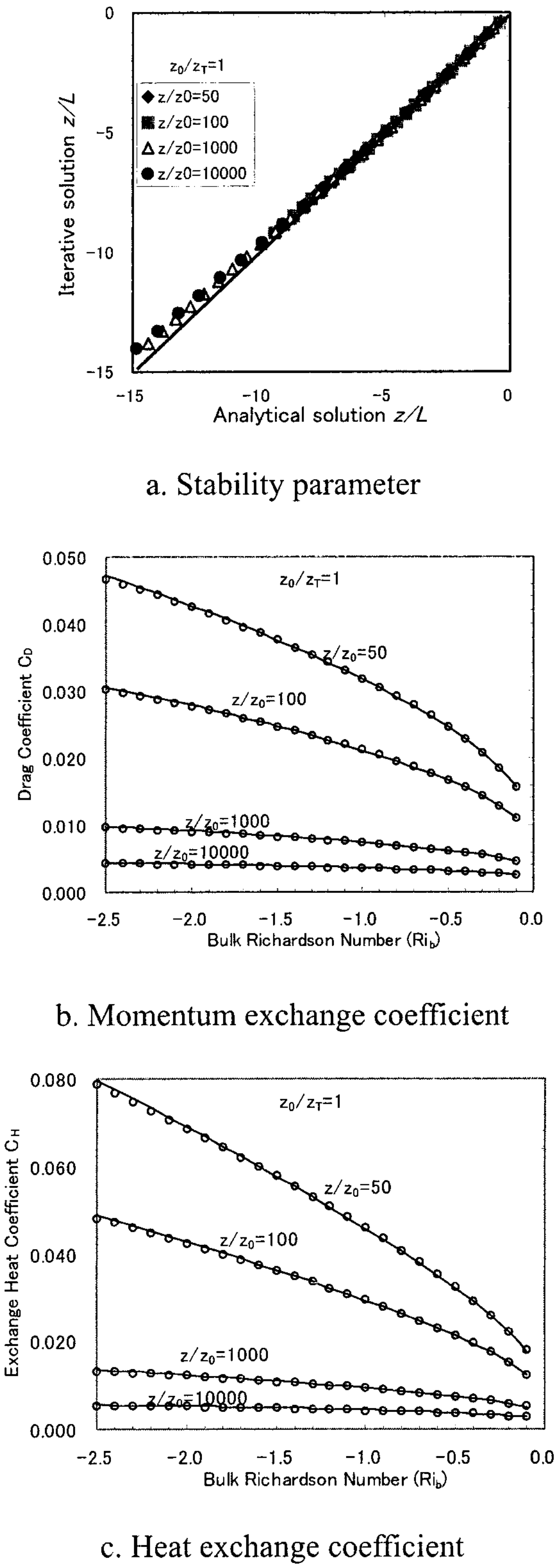
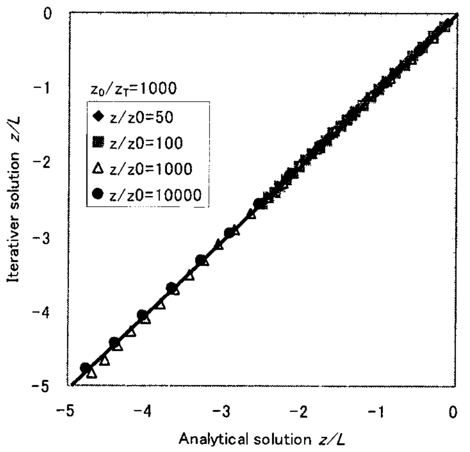
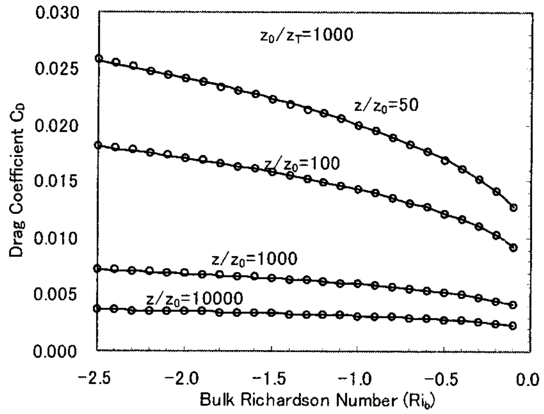


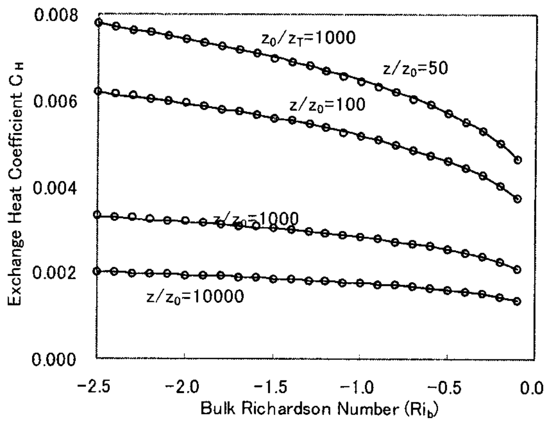
FIG. 2. Comparison between the analytical solution and the iterative solution for $z_0 = z_T$. (a) The horizontal value is ξ from Eq. (13) and the vertical value is the iterative solution. (b), (c) The lines are for the analytical solution; the dots are for the iterative solution. Profile functions are from Högström (1996).



a. Stability parameter



b. Momentum exchange coefficient



c. Heat exchange coefficient

FIG. 3. Same as Fig. 2, but for $z_0/z_T = 1000$.

q represent the values at the reference level z, and ρ is the air density. Parameters C_D and C_H are bulk transfer coefficients or exchange coefficients:

$$C_D \equiv \frac{u_*^2}{U^2} = k^2[\ln(z/z_0) - \psi_m(\xi, \xi_0)]^{-2}, \quad \text{and} \qquad (16a)$$

$$C_H \equiv \frac{u_*\theta_*}{U(\Theta - \Theta_s)} = \frac{k^2}{\mathrm{Pr}_0}[\ln(z/z_0) - \psi_m(\xi, \xi_0)]^{-1} \times [\ln(z/z_T) - \psi_h(\xi, \xi_T)]^{-1}. \qquad (16b)$$

Given U, Θ, and Θ_s, the stability parameter ξ can be obtained from Eq. (8) or Eq. (13), and then C_D and C_H are calculated through Eqs. (16a) and (16b). Because Eq. (8) is the exact solution over the moderately stable range, we only investigate the performance of the approximate solution Eq. (13) in the unstable range.

In the case of $z_0 = z_T$, the comparison between the analytical solution of Eq. (13) and the iterative solution is shown in Figs. 2a–c for the stability parameter ξ, the momentum exchange coefficient C_D, and the heat exchange coefficient C_H, respectively. Figure 2a shows that the analytical solution is very close to the iterative solution. Only when $\xi < -10$, does the analytical one deviate from the iterative one. However, the difference in ξ does not lead to appreciable errors of C_D and C_H in this case. Figures 2b,c indicate that the analytical solution of exchange coefficients agrees well with iterative solution. The relative errors of C_D and C_H are no more than 3%.

For surfaces with bluff elements, such as villages and urban areas, z_T may be very small when compared with z_0. In the case of $z_0/z_T = 1000$, the comparisons in ξ, C_D, and C_H between the approximate analytical solution and numerical iterative solution are shown in Figs. 3a–c. Again, the analytical solution shows a good agreement with the iterative solution. The relative errors of C_D and C_H are no more than 1.5%.

In addition, the analytical solution was compared with the iterative solution in the cases of $z_0/z_T = 10$ and $z_0/z_T = 10^4$, which again shows the high accuracy of the analytical solution. The computation in the range $z_0/z_T = 1$–10^4 and $z/z_0 = 50$–10^4 further indicates that error in the analytical solution usually decreases with the increase of z/z_0.

The above investigation was based on Högström's (1996) profile functions. Using the other profile forms in Table 1 and the corresponding coefficients in Table 2, we obtain similar accuracy to the above.

5. Solution comparison in the case of $z_0 = z_T$

As mentioned before, various approximate analytical solutions for $z_0 = z_T$ have been studied and applied in numerical models. Two simple but effective analytical

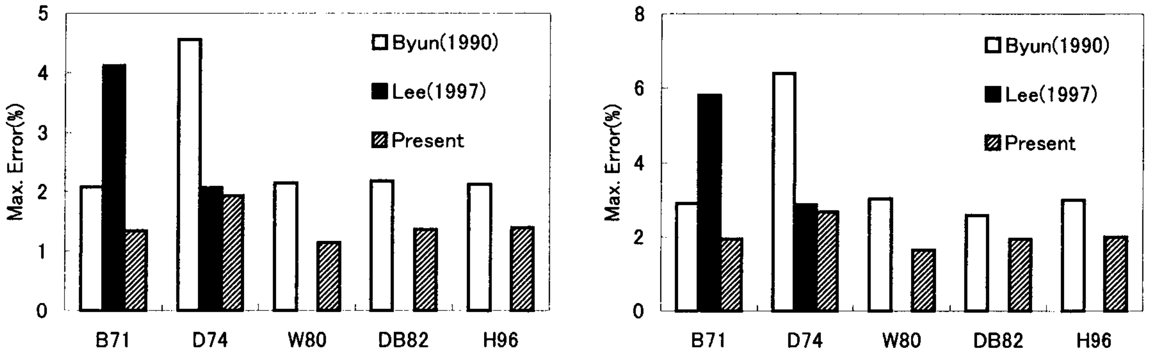


FIG. 4. The maximum errors (%) of (left) drag coefficient and (right) heat exchange coefficient of three analytical solutions for $z_0 = z_T$. The abbreviations are defined in Table 1.

solutions were proposed by Byun (1990) and Lee (1997). Byun's solution automatically includes the coefficients of profile functions, and its expression is independent of profile functions; Lee's solution contains two coefficients related to the profile form, and the coefficients for the profile forms of Businger et al. (1971) and Dyer (1974) have been determined by Lee. The following compares the two previous solutions with the current analytical solutions in the case of $z_0 = z_T$.

Figure 4 shows the maximum error of each solution in exchange coefficients in the range of $z/z_0 = 50–10^4$ and $\mathrm{Ri}_b \geq -2.5$. Byun's solution shows a high accuracy for all profile forms in Table 1 except Dyer's form; Lee's solution performs well with Dyer's form but less so with Businger et al.'s form. However, the maximum error of the current solution for all profile forms is no more than 2% in drag coefficient and 3% in heat exchange coefficient, less than that of the two previous solutions.

6. Conclusions

The Monin–Obukhov similarity theory provides the most suitable and acceptable framework for a quantitative description of the mean and turbulence structure of a stratified surface layer. Its application requires the solution of the stability parameter equation. This note has addressed its analytical solution when the thermal roughness length differs from the aerodynamic roughness length. For a moderately stable surface layer, the solution based on linear profile form is exact; for an unstable surface layer, an approximate analytical solution is proposed to improve computational efficiency. The solution contains several coefficients that depend on the form of profile functions. The coefficients corresponding to five typical forms are given. Even for a new form, the coefficients can be easily fitted with the iterative solution.

The stability parameter and turbulent exchange coefficients of the approximate solution and iterative solution were compared in the range of $z/z_0 = 50–10^4$ and $z_0/z_T = 1–10^4$. The approximate ξ is close to the iterative solution except when $-\xi$ is very large. The exchange coefficients for momentum and heat agree well with those computed using a numerical iterative method. A comparison between the current solution and two previous solutions was also carried out for the special case of $z_T = z_0$, and it indicates that the current solution is more accurate than the other two.

Given the uncertainties in determining variables such as surface radiometric temperature, the error of the current analytical solution is acceptable for practical application. This study provides a simple and effective analytical solution of similarity equations that may be applied to modeling the interaction between the land surface and the atmosphere.

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TABLE 2. Coefficients in Eq. (14) corresponding to five typical forms of profile functions.

Profile form*	c_{000}	c_{100}	c_{010}	c_{001}	c_{110}	c_{011}	c_{101}	c_{200}	c_{020}	c_{002}
B71	0.076	−0.108	−0.296	0.335	0.053	0.184	−0.026	0.017	−0.073	−0.132
D74	−0.172	−0.027	−0.622	0.837	0.127	0.377	−0.122	0.014	−0.134	−0.296
W80	0.042	−0.095	−0.265	0.310	0.051	0.172	−0.025	0.017	−0.068	−0.124
DB82	0.052	−0.088	−0.190	0.214	0.039	0.123	−0.013	0.015	−0.049	−0.088
H96	0.048	−0.099	−0.292	0.340	0.054	0.189	−0.028	0.018	−0.075	−0.136

* See Table 1 for source explantations.

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